



A NOVEL TECHNIQUE FOR INVERSE IDENTIFICATION OF DISTRIBUTED STIFFNESS FACTOR IN STRUCTURES

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A computational inverse technique for identifying stiffness distribution in structures is proposed in this paper using structural dynamics response in the frequency domain. In the present technique, element stiffness factors of the finite element model of a structure are taken to be the parameters, and explicitly expressed in a linear form in the system equation for forward analysis of the harmonic response of the structure. This offers great convenience in applying Newton's method to search for the parameters of stiffness factor inversely, as the Jacobian matrix can be obtained simply by solving sets of linear algebraic equation derived from the system equation. Examples of identifying stiffness factor distribution which is often related to damage in the elements of the structure are presented to demonstrate the present technique. The advantages of the present technique for inverse parameter identification problem are (1) the number of the parameters can be very large; (2) the identification process is very fast and (3) the accuracy is very high. The efficiency of the proposed technique is compared with genetic algorithms.

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1. INTRODUCTION

Identification of parameters of structural systems in engineering is of considerable importance in practice due to the increasing demands for assessment of integrity and reliability of structures. Non-destructive testing techniques used for this purpose are based on the measurement of the dynamic characteristics of structures, i.e., dynamic responses, modal parameters and wave scattering characteristics of the structures [1, 2]. These dynamic characteristics of structures are related to the structural parameters in a forward relationship established using analysis models. That is, for given parameters these dynamic characteristics can be estimated through a mathematical model in forward analysis. Explicit inverse relationship can only be found in some specific cases. In general, it is impossible to find explicit inverse relationship. Thus, a computational technique is often required to solve inverse problems to reconstruct the structural parameters based on a given model. However, the inverse analysis is much more difficult in comparison with the forward analysis, because of its non-linear and ill-posed nature of the problem.

Considerable work has been done in the area of structural parameters identification based on inverse computational techniques, especially for parameters which are difficult or even impossible to be measured accurately using traditional experimental techniques. These structural parameters include material constants of anisotropic material, constraints stiffness of the boundary, flaw and/or cracks involved in structures during serving and manufacturing, etc. Liu and Han [3] have inversely determined material properties of

functionally graded material which is difficult to be experimentally determined. Balasubramaniam and Rao [4] and Liu *et al.* [5] have reconstructed material constants of homogeneous laminated composite materials using elastic wave responses. The anisotropic material constants of composite material can also be inversely identified using structural vibration properties such as modal parameters [6, 7]. Using elastic wave scattering in structures, it is also possible to inversely determine cracks in a composite laminates [8, 9]. In determination of flaw and damage in structures, the damage parameters are generally related to the stiffness reduction as discussed by Araújo dos Santos *et al.* [10], Bicanic and Chen [11] and Chen and Bicanic [12]. Discretizing the structure with a number of finite elements, the stiffness distribution in the structure can thus be expressed by the element stiffness factor or damage factor. In solving this type of problems, the difficulty is the large number of parameters. Araújo dos Santos *et al.* [10] derived a sensitivities matrix of the eigenvalues with respect to the damage factors of element from the orthogonality condition of the mode shapes. Thus, the element stiffness factors can be solved through a set of linear equation using the measurements of natural frequencies and mode shapes. With no previous knowledge of damaged areas and locations, this method allows the identification of multiple damages when enough modal parameters are known. Bicanic and Chen [11] proposed a procedure for the damage identification of framed structures using only a limited number of measured natural frequencies. Based on the characteristic equation of the original and damaged structure, a set of equations is formulated corresponding to a difference change in the stiffness matrix, and it is solved by the direct iteration and Gauss–Newton techniques. In general, when solving an inverse problem of structural parameter identification, it is simply formulated as an objective function given by a weighted sum of squared differences between the measured data and the corresponding simulated value of the dynamic properties of structures. With this formulation, the inverse reconstruction can be solved by means of optimization methods to minimize the objective function. As the complexity of the objective function, genetic algorithms (GAs) have been widely used as a searching technique for such difficult and non-linear problems without sensitivity analysis and initial guess [4, 7, 13]. Another very important advantage of GAs is the convergence property to the global optimal of the solution. However, it is computationally extensive and it suffers from slow convergence rate at later stage due to the nature of random searching. For problems with large number of parameters to be identified, use of GA becomes impractical, especially when the forward analysis is time consuming.

In this study, an error function is defined in a form of nonlinear implicit equations of unknown parameters which give the difference between numerically predicted results and measured values of the harmonic response of structures. Newton's method is applied iteratively to search for the parameters which is the solution of the root of the error function. With a finite element model of a structure, the simulated harmonic response at certain frequency is computed in forward analysis and the Jacobian matrix can also be obtained by solving a set of linear equations derived from the forward system equation. It is found that the proposed method can be used to solve problems with large number of parameters to be identified. Examples and comparison with a GA-based search scheme demonstrate the high efficiency and excellent accuracy of the proposed technique.

2. FINITE ELEMENT FORMULA IN FORWARD ANALYSIS

In this study, we consider a general finite element model of a linear-elastic structure. The dynamic governing equation is given by

$$[M]\{\ddot{d}\} + [K]\{d\} = \{F\}, \quad (1)$$

where $[K]$ and $[M]$ are global stiffness matrix and mass matrix, respectively, $\{d\}$ is the global nodal displacement vector and $\{F\}$ the nodal load vector. With this equation, the structure dynamic properties such as responses and modal parameters can be obtained for given stiffness, mass matrix and load vector.

For a harmonic excitation $\{F\} = \{P\}e^{i\omega t}$, the harmonic displacement response can be written as

$$\{d\} = \{u\}e^{i\omega t}, \quad (2)$$

where $\{u\}$ is the vector of displacement amplitude. Equation (1) for harmonic excitation can be written as

$$\{[K] - \omega^2[M]\}\{u\} = \{P\}. \quad (3)$$

For modal properties analysis, the characteristic equation used to determine the modal parameters of the structure can be written as

$$([K] - \lambda_\alpha[M])\{\phi_\alpha\} = 0, \quad (4)$$

where λ_α and $\{\phi_\alpha\}$ are the α th eigenvalue and the corresponding eigenvector (mode shape) of the structure.

The global stiffness matrix of a structure is an assembly of the elements' stiffness matrix, and for isotropic elastic material, the element stiffness matrix is always proportional to the elastic modulus of the material and the geometric coefficient, which are unknown parameters in an inverse analysis. Thus the global stiffness is expressed as

$$[K] = \sum_{i=1}^N x^i [K_i]^e, \quad (5)$$

where N is the total number of elements, $x^i (i = 1, N)$ the unknown parameters of elastic modulus or element stiffness factor and the element stiffness $[K_i]^e$ is obtained by assuming a unit factor. In general, the element stiffness factor $x^i (i = 1, N)$ reflects the degree of damage in the element of a damaged structure. Substituting equation (5) into equations (3) and (4), we have

$$\left\{ \sum_{i=1}^N x^i [K_i]^e - \omega^2 [M] \right\} \{u\} = \{P\} \quad (6)$$

and

$$\left\{ \sum_{i=1}^N x^i [K_i]^e - \lambda_\alpha [M] \right\} \{\phi_\alpha\} = 0. \quad (7)$$

Note that ω and $\{P\}$ are known for the given excitation force. Mass matrix $[M]$ is known for the given material for the structure. $[K_i]^e$ is also known once the finite element mesh is given. For an assumed set of x^i , $\{u\}$ can therefore be computed without any difficulty and so can the modal parameters.

3. INVERSE IDENTIFICATION FORMULATION

In Forward analysis, the displacement response and modal parameters of a finite element system can be predicted using equations (6) and (7) with given parameters $x^i (i = 1, N)$. However, in inverse analysis, the parameters are needed to be identified using the measured value of the displacement response or modal parameters. That is, parameters are chosen

such that they fit the experiment data. There are two methods to fit these data. One is simply using the least-squares method which minimizes the square error sum; the other is the sensitivity based analysis method which has different formulation for different problems considered and it is often obtained approximately, neglecting the second order of the variation.

3.1. OBJECTIVE FUNCTION

The commonly used objective function is defined using the weighted sum of squared differences between the measured data and the corresponding simulated value of the dynamic properties of structures.

$$E(\mathbf{x}) = \sum_{i=1}^l W_i (f_i(\mathbf{x}) - f_{i0})^2, \quad (8)$$

where l is the total number of measurements, \mathbf{x} is the vector of unknown parameters $(x^1, x^2, \dots, x^N)^T$, f_{i0} is the measured value and $f_i(\mathbf{x})$ the corresponding simulated value for a trial \mathbf{x} and W_i the weight factor. The measured values of a structure can be the responses, natural frequencies and the values of modal assurance criterion (MAC) [14] that is related to the mode shapes.

3.2. PROPOSED DIRECT FORMULATION

Sensitivity-based formulation derived from modal parameters has been used to solve the parameters, which is a little complicated and often approximate. Here a formulation is proposed using harmonic response. For a finite element model with n elements, n displacements at different nodes on the structure can be measured, and expressed in a vector form of $\{\bar{u}\}$. The identification problem is to determine the element stiffness factor vector \mathbf{x} in equation (6) using the measured response $\{\bar{u}\}$. That is, to find \mathbf{x} that satisfies

$$[Q]\{u\} = \{\bar{u}\}, \quad (9)$$

where $[Q]$ is a constant matrix with elements of zeros or ones, which selects the degrees of freedom corresponding to the measured displacement components. Vector $\{u\}$ is solved from equation (6) for a given \mathbf{x} .

Define an error function of

$$\mathbf{f}(\mathbf{x}) = [Q]\{u\} - \{\bar{u}\} = 0, \quad (10)$$

where

$$\mathbf{f}(\mathbf{x}) = \begin{Bmatrix} f_1(x^1, x^2 \dots x^n) \\ f_2(x^1, x^2 \dots x^n) \\ \vdots \\ f_n(x^1, x^2 \dots x^n) \end{Bmatrix}, \quad (11)$$

$$\mathbf{x} = (x^1, x^2 \dots x^n)^T.$$

Here, $\mathbf{f}(\mathbf{x})$ is a set of non-linear implicit equation with respect to the parameters. The value of $\mathbf{f}(\mathbf{x})$ and its derivation can be evaluated through equation (6). Thus, the solution of equation (10) can be solved directly using Newton's method numerically.

4. INVERSE COMPUTATIONAL TECHNIQUE

4.1. OPTIMIZATION METHOD

Based on the objective function, the identification problem can be solved by optimization techniques to search for the parameters which minimize the objective function. Classical gradient technique such as the least-squares optimization method and direct search scheme like genetic algorithms (GAs) has been used to find parameters which minimize the objective function.

4.2. NEWTON'S METHOD

For the proposed error function, Newton's method is used directly to solve the nonlinear system for the parameters. Newton's method uses an iterative process to approach a root of a function $f(x)$. Beginning with an initial trial value of x_0 , the succeeding solution is obtained through

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \quad (12)$$

where x_k is the solution obtained in the previous iteration, $f(x_k)$ and $f'(x_k)$ represent the value of the function and its derivative at x_k , respectively, and x_{k+1} the current iteration result. When x_k converge to a value, it will be a root of the function.

For non-linear equation system $\mathbf{f}(\mathbf{x}) = 0$ ($f_i(x^1, \dots, x^n) = 0, i = 1, n$), in the \mathbf{R}^n , a similar iteration formula is given below

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \mathbf{J}^{-1}(\mathbf{x}_k) \mathbf{f}(\mathbf{x}_k), \quad (13)$$

where $\mathbf{J}(\mathbf{x})$ is the Jacobian matrix of the system equations given below

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x^1} & \frac{\partial f_1}{\partial x^2} & \cdots & \cdots & \frac{\partial f_1}{\partial x^n} \\ \frac{\partial f_2}{\partial x^1} & \frac{\partial f_2}{\partial x^2} & \cdots & \cdots & \frac{\partial f_2}{\partial x^n} \\ \vdots & \vdots & & & \vdots \\ \frac{\partial f_n}{\partial x^1} & \frac{\partial f_n}{\partial x^2} & \cdots & \cdots & \frac{\partial f_n}{\partial x^n} \end{bmatrix}$$

and \mathbf{x}_k is the solution obtained in the previous iteration, $\mathbf{f}(\mathbf{x}_k)$ and $\mathbf{J}(\mathbf{x}_k)$ represent the value of the functions and its Jacobian matrix at \mathbf{x}_k , respectively, and \mathbf{x}_{k+1} the current iteration result. Starting with an initial guess \mathbf{x}_0 , equation (13) is expected to converge to a solution of equation (10). The iteration stops when specified accuracy reached:

$$\|\mathbf{x}_{k+1} - \mathbf{x}_k\| \leq \varepsilon.$$

It is to be noted that to ensure equation (13) is determined, the number of measurements should equal the number of parameters. In this case, Newton's method can get the solution very fast if it converges. However, it has the local convergence properties and may not converge or converge to values which exceed the physically defined validity region when

started from certain initial guess. To improve the performance of Newton's method while remaining in the fast convergence rate, a modification is made to correct the iteration stepsize when necessary.

$$\mathbf{x}_{k+1} = \mathbf{x}_k - (\mathbf{J}(\mathbf{x}_k) + \Delta^k)^{-1} \mathbf{f}(\mathbf{x}_k), \quad (14)$$

where Δ^k is a diagonal matrix, it is chosen such a way as to ensure that $\|\mathbf{f}(\mathbf{x})\| \rightarrow 0$ and make the solution converge.

To ensure that the solution falls into the physically feasible region, an upper and lower bound is applied to constrain the parameters:

$$\mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u.$$

Here \mathbf{x}_l and \mathbf{x}_u are the lower and upper bounds respectively.

5. CALCULATION OF JACOBIAN MATRIX

The Newton's method requires the calculation of Jacobian matrix, the derivatives of displacements with respect to the unknown parameters, element stiffness factors \mathbf{x} . The Jacobian matrix can be obtained efficiently by taking advantage of the linear expression of x^i in equation (6). Performing differentiations on both sides of equation (6) with respect to each parameter x^i leads to

$$[K_i]^e \{u\} + \{[K] - \omega^2[M]\} \left\{ \frac{\partial u}{\partial x^i} \right\} = 0 \quad (i = 1, N). \quad (15)$$

In equation (15), vector $\{u\}$ is solved from equation (6) in forward analysis. So equation (15) can be written as

$$\{[K] - \omega^2[M]\} \left\{ \frac{\partial u}{\partial x^i} \right\} = -[K_i]^e \{u\} \quad (i = 1, N). \quad (16)$$

Thus, the derivative $\{\partial u / \partial x^i\}$ can be solved from the above linear algebraic equation system which is in the same form as equation (6). For $i = 1, N$, the Jacobian matrix is obtained by multiplying matrix $[Q]$.

6. PROCEDURE OF ITERATION

Starting with an initial guess \mathbf{x}_0 , the procedure of iteration is given as follows:

Step 1: Solve equation (6) at \mathbf{x}_k for $\{u\}$ and then compute the value of error function

$$\mathbf{f}(\mathbf{x}_k) = [Q] \{u\} - \{\bar{u}\}. \quad (17)$$

Step 2: Solve equation (16) at \mathbf{x}_k for $\{\partial u / \partial x^i\}$ and obtain the Jacobian matrix. In solving equation (16), the right side vector is formed as "a pseudo-load vector" first by using the response obtained above, the coefficient matrix has been factorized in Step 1 for forward analysis, the derivation vector is then obtained by back-substitution of the pseudo-load vector.

Step 3: Find \mathbf{x}_{k+1} by Newton's method using equation (13). In practice, \mathbf{x}_{k+1} is obtained by solving the linear equation system

$$\mathbf{J}(\mathbf{x}_k)(\mathbf{x}_{k+1} - \mathbf{x}_k) = \mathbf{f}(\mathbf{x}_k) \quad (18)$$

Step 4: Repeat Steps 1–3 until $\|\mathbf{x}_{k+1} - \mathbf{x}_k\| < \text{tolerance}$.

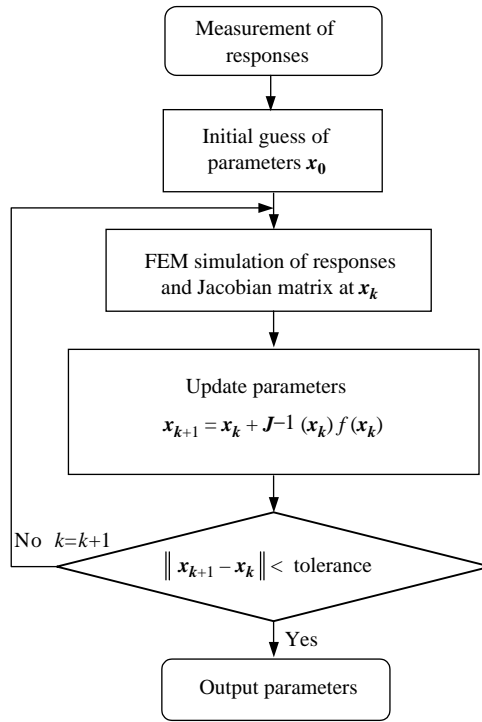


Figure 1. Flowchart of the procedure.

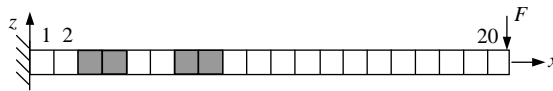


Figure 2. Non-uniform stiffness beam and its finite element model.

Figure 1 shows the flowchart of the procedure.

7. EXAMPLES AND DISCUSSION

7.1. CANTILEVER BEAMS

In order to verify the proposed technique, the cantilever beam shown in Figure 2 is considered. It is discretized into 20 beam elements. Hence, there are 20 unknown parameters that represent the stiffness factors of elements. It is related to the material constant and/or second moment of the section area. The element stiffness factor to be identified is given in the tables. The mass density is $\rho = 7.8 \times 10^3 \text{ kg/m}^3$, and the second moment of section area is $I_z = 0.8 \times 10^{-8} \text{ m}^4$. The excitation is a time harmonic load at the free tip of the beam with a frequency of $\omega = 100 \text{ rad/s}$. The measured deflection amplitude at 20 nodes is simulated using computational analysis results for the given true parameters.

In case 1, a piecewise uniform stiffness distributed beam is considered, and the true stiffness factors are given in Table 1. We start the iteration from an initial guess x_0 that takes

TABLE 1
Element stiffness factors

Element number	1-5	6-10	11-15	16-20
Stiffness factor	2·1	1·5	2·1	1·5

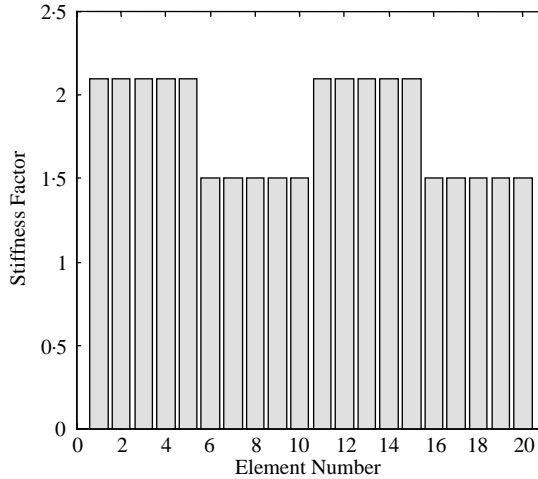


Figure 3. Stiffness distribution of elements (case 1).

TABLE 2
Element stiffness factors

Element number	1-2	3-4	5-6	7-9	10-20
Stiffness factor	2·1	1·8	2·1	1·05	2·1

uniform value of 2·1 (undamaged stiffness factor) for all the parameters. It converges to the solution very fast, and the results are shown in Figure 3. The results are in very good agreement with the true values given in Table 1. The same results can be obtained for different values of ω .

In case 2, a damaged beam with two damaged locations is considered. The true stiffness factors are given in Table 2. The damage factor β_f^i of the i th element is defined as the deduction of the element stiffness, and can be obtained from the stiffness factor.

$$\beta_f^i = \left(1 - \frac{x^i}{x} \right), \tag{19}$$

where x represents the value of undamaged stiffness factor. The damage in elements 3, 4 and 7-9 is successfully detected as shown in Figure 4.

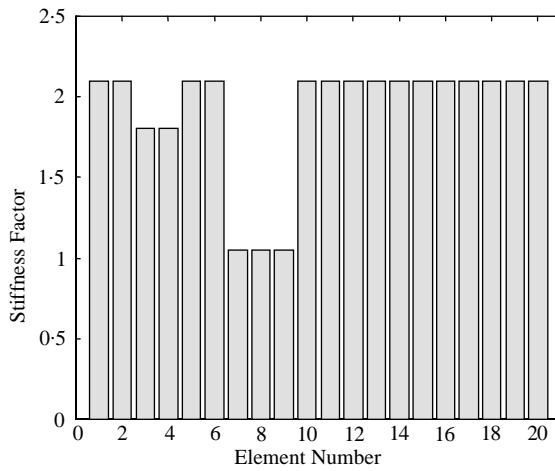


Figure 4. Stiffness distribution of elements (case 2).

TABLE 3
Element stiffness factors

Element number	1-2	3	4	5	6-20	21-25	26-50
Stiffness factor	2.1	1.8	1.5	1.2	2.1	1.5	2.1

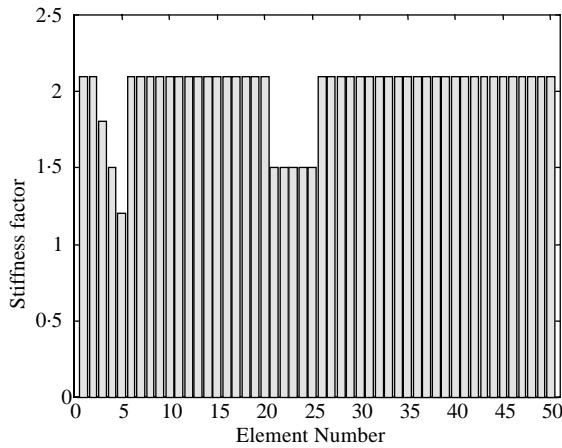


Figure 5. Stiffness distribution of elements (case 3).

In case 3, the same beam is now discretized into 50 elements and the stiffness distribution is represented by 50 parameters. Table 3 shows the true element stiffness factor. The results are obtained very fast and accurate as shown in Figure 5. The results also indicate clearly the damage status or the stiffness distribution of the beam in terms of the stiffness factor.

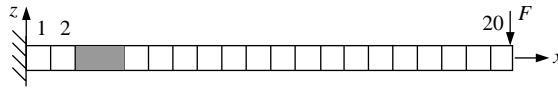


Figure 6. Beam with one damage location.

The examples have shown that the proposed inverse technique is suitable for problems with large number of parameters to be identified. It takes only seconds of CPU time to obtain a high accurate result for the beam structure considered. It can be applied to damage detection problems involving multiple distributed defects with arbitrary degree of damage accurately. However, like the gradient-based optimization algorithm, the initial guess may affect the iteration progress. For suitable initial guesses, identical results can be obtained. It should also be pointed out that the frequency of the exciting force could not be close to the natural frequency of the structure in case equation (16) becomes singular and the inverse procedure fails because of no damping terms being considered.

7.2. COMPARISON STUDY WITH GAS

In order to compare the performance of the proposed technique with genetic algorithm, the same beam as shown in Figure 1 is considered. A micro-GA (μ GA) with an elitist strategy has been applied to the stiffness factor identification (damage detection). The population is taken to be 5 in each generation while the probability of uniform crossover is set to be 0.5. The objective function (8) is employed for fitness evaluation with harmonic response of deflection taken as input. The stiffness factors of 20 elements are taken as the parameters to be identified. In GAs, these parameters are required to be discretized according to the accuracy needed. When all the parameters are discretized into 8 grades in the range of 0.63–2.1, the discrete search space contains 2^{60} candidates. It is found that for such a great number of possibilities, the CPU time spent is excessively long due to the random nature of GAs and the time consumed in the forward analysis. In order to decrease the number of parameters, the beam including one damage location with only 1–4 elements damaged is assumed. The damage degrees of these elements are discretized into 8 grades. This decreases the discrete search space to contain 2^9 candidates and makes the search efficiently. As an example, the beam including one damage location shown in Figure 6 is considered. The damage is located in elements 3 and 4 with stiffness deduction factor $\beta_f = 0.5$. It takes 30 generations to obtain the solution. The CPU time consumed is 40 s. When the proposed technique is employed to the same problem, it takes only about 1.5 s. This verified the efficiency of the proposed technique over GAs for problems with multiple continue variables as parameters and analytical to the variables. Another advantage of the proposed technique over GAs is that the GAs' accuracy is limited to the possibilities to discretize the parameters. In order to increase the accuracy of GAs, more possibilities are required to discretize the parameters and more generations are required to search for the solution.

7.3. PLATES

The technique developed here is general; it can be easily extended to other types of finite element structures. The following example shows its application to plate structures. The

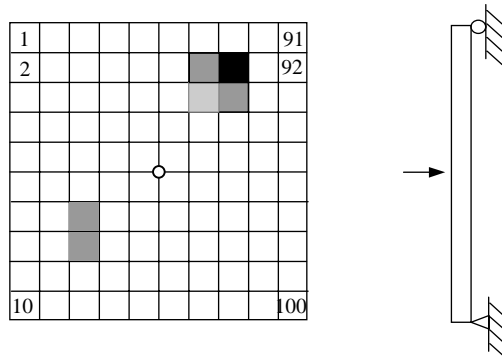


Figure 7. Simply supported square plate.

TABLE 4

Element stiffness factors

Element number	27–28	52	53	62	63	The rest
Stiffness factor	1.5	1.6	1.8	1.2	1.5	2.1
Damage factor	0.286	0.238	0.143	0.429	0.286	0

simply supported plate with several elements deducted in stiffness is given in Figure 7. It is divided into 100 elements and simulated using eight-noded isoparametric quadratic plate element. Nodal deflection response under harmonic excitation is taken as the measurements which is simulated numerically. The true element stiffness factor is given in Table 4. Once again the stiffness factors of the elements are identified accurately as shown in Figure 8. In Figure 8, a damage factor is employed instead of a stiffness factor.

7.4. CANTILEVER BEAM WITH MEASUREMENT ERROR

The efficiency and accuracy of the proposed technique has been demonstrated through the above examples without considering the random measurement errors. To study the effect of the random measurement errors on the parameters identification, a normally distributed error with zero mean and constant standard deviation is added to the measured value. Taking the beam considered in case 2 as an example, we considered several cases of randomly produced error with the same zero mean and same deviation. When adding these errors to the measured responses, respectively, only in some cases we can obtain good result as shown in Figure 9. In some cases, we fail to get any results. This means that the proposed technique is error sensitive because of no regularization.

In our study, we proposed an inverse procedure-based numerical method. It has been verified analytically. However, to apply the proposed method to solve practical problems, some consideration and modification are required. First of all, the forward analysis model of structural system should be carefully considered to simulate the practical system as accurately as possible or correction to the difference between simulated and practical responses should be made. For example, damping terms and supports stiffness of boundary should be considered. Another important problem considered is the measuring error. For

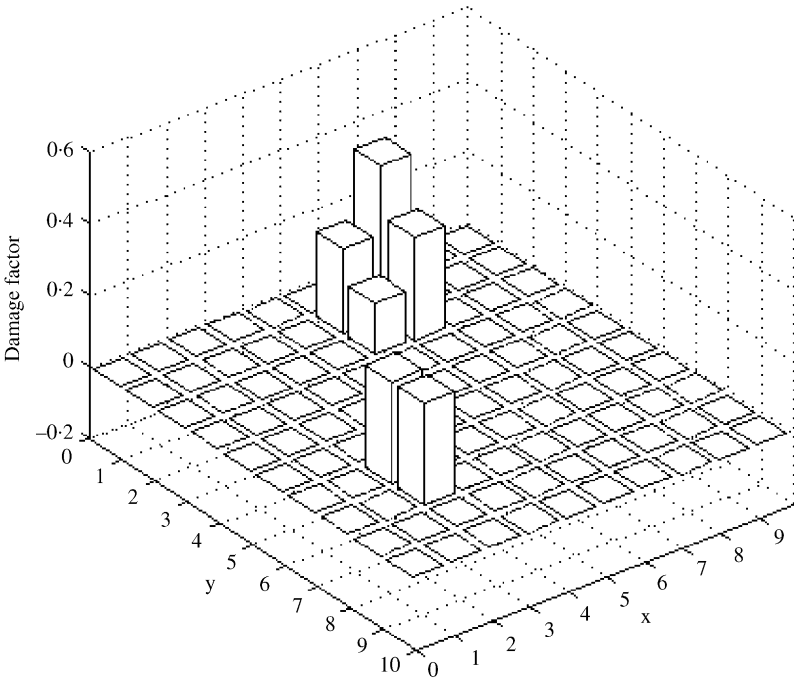


Figure 8. Damage factor identified.

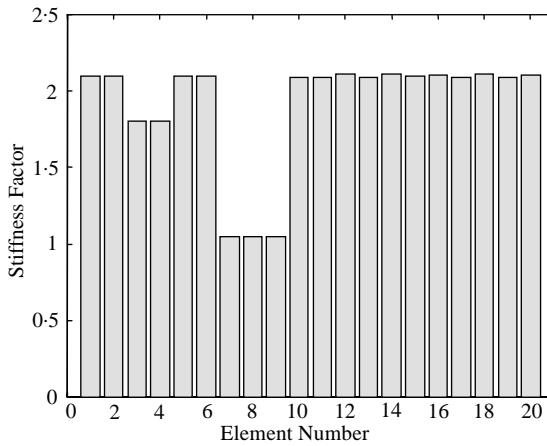


Figure 9. Stiffness distribution of elements (with measurement error).

considering measurement error, Gauss–Newton method should be used instead where the number of measured data is more than the number of parameters. Gauss–Newton method gives the estimation of the parameters based on the minimization of the least squares of the error norm. It is expected to be robust to random errors of measurement.

8. CONCLUSION

A novel inverse technique has been proposed and implemented for the identification of distributed stiffness factor and damage parameters in a structure. The measurement of

harmonic response to the excitation is used in the present technique. The proposed method based on a set of implicit equations, and Newton's iteration method is applied to find the solution which fits the measured data with predicted result. It converged to the true solution much faster in comparison with random-based GAs. Examples verified the accuracy and efficiency of the proposed method for problems with large number of parameters to be identified. However, improvements are required to apply it to practical problems.

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